DETERMINATION OF THE PARAMETERS OF A SUPERSONIC GAS JET IMPINGING ON AN INCLINED FLAT OBSTACLE

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Results are given of a theoretical and experimental investigation of the interaction of a supersonic gas jet issuing from a conical nozzle with an inclined flat obstacle. Relations have been obtained for determining the boundaries of the spreading of the stream around the obstacle and the gasdynamic parameters. The results of the computation are confirmed by experiment.

An account is given of an approximate method for determining the gasdynamic parameters of a supersonic stream incident on an obstacle, based on the results obtained in experiments. The experimental investigation included the study of the spread of a gas jet over a flat obstacle, and also the measurement of static pressures on it. The tests were carried out in a supersonic wind tunnel operated by compressed air ($k = 1.405$). The supersonic jet was formed by conical nozzles having the following geometrical and gasdynamic parameters: $d_a = 20$ mm, $\vartheta_a = 5°$, $M_a = 1.5$, 2.0, 2.5, and 3.0.

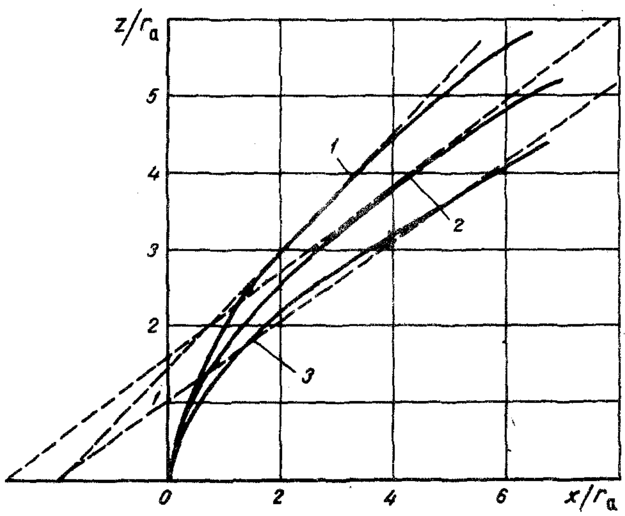

Fig. 1. Boundary of spreading of the gas jet over an obstacle (solid lines—experiment, dashed—theory): 1) for M = 1.5, n = 10.8; 2) M = 2.0, n = 7.6; 3) M = = 1.0, n = 5.1.

A flat steel obstacle (400 × 350 × 10 mm) was attached to a traverse mechanism in the wind tunnel working section. The traverse mechanism allowed variation of the angle of inclination of the obstacle to the nozzle axis φ and of the distances from the nozzle exit to the obstacle. To study the distribution of the pressure of the gas jet on the obstacle, holes were drilled in the latter (of diameter 0.7 mm, and with pitch 4 mm). To allow visualization of the spreading out of the jet over the obstacle, the latter had fastened on it a transparent plastic sheet of thickness 10 mm. By putting a thin layer of a viscous substance, e. g., lubricating grease, mixed with powdered graphite

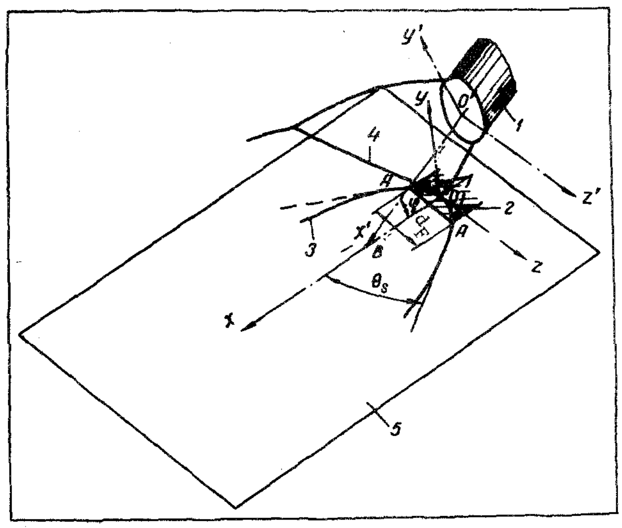

Fig. 2. Schematic of the interaction of a gas jet with an inclined flat obstacle: 1) nozzle; 2) initial point of interaction of the jet with the obstacle; 3) lines of spreading of the jet over the obstacle; 4) oblique shock; 5) obstacle.

ahead of the injection point on the plastic surface, we were able to establish the relationship between the angle of spreading and the angle of slope of the obstacle φ, the Mach number at the nozzle exit, the distance from the nozzle exit to the obstacle, and the off-design factor for the jet discharge $n = P_a/P_e$.

Analysis of the experimental data showed that the angle of spreading of the jet over the flat obstacle may be calculated with sufficient accuracy from the Prandtl-Meyer formula for plane expansion flow (Fig. 1). In doing this the parameters of the undisturbed flow should be taken as those of the gas behind the oblique density discontinuity which is formed at the point where the jet meets the obstacle, and it should be assumed that the expansion occurs up to atmospheric pressure. We put forward the following method for calculating the parameters of the interaction of the supersonic jet with the obstacle, based on the good agreement between experimental and calculated data on the angle of spreading.

We determine the parameters at the boundary of the jet discharging from the nozzle: Mach number from the relation

$$M_e = \sqrt{\frac{2}{k-1}\left[\left(1+\frac{k-1}{2}M_a^2\right)\left(\frac{P_e}{P_a}\right)^{\frac{1-k}{k}}-1\right]}, \quad (1)$$

density, temperature, and the other gasdynamic parameters—from the known formulas for isentropic

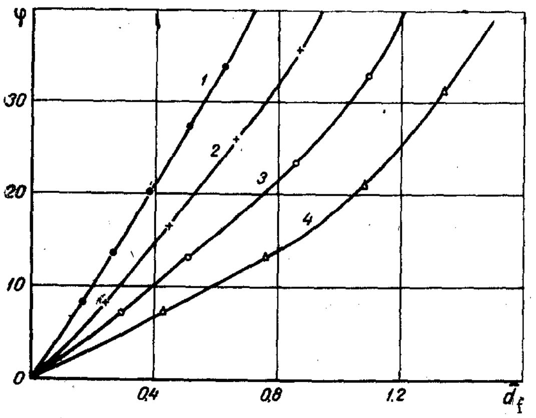

Fig. 3. Diameter of the fictitious nozzle $\bar{d}_f$ as a function of angle of inclination of the obstacle to the jet axis φ, and of the angle of spreading of the stream over the obstacle θ_s: 1) $\theta_s = 60°$; 2) 50°; 3) 40°; 4) 30°.

gas flow [1]. For the initial angle of inclination of the theoretical jet boundary we have

$$\theta_e = \vartheta_a + \omega(M_e) - \omega(M_a), \tag{2}$$

where

$$\omega(M) = \sqrt{\frac{k+1}{k-1}} \operatorname{arctg} \sqrt{\frac{k-1}{k+1}(M^2-1)} - \operatorname{arctg}\sqrt{M^2-1}. \tag{3}$$

Knowing the jet parameters at the boundary and the angle of inclination of the obstacle to the jet axis, we can determine, from the formulas of oblique shock theory, the angle of slope β of the latter, and the parameters behind it (Fig. 2):

$$\operatorname{ctg}(\theta_e + \varphi) = \left[1 + M_e^2\left(\frac{k+1}{2} - \sin^2\beta\right)\right] \operatorname{tg}\beta \Big/ (M_e^2 \sin^2\beta - 1), \tag{4}$$

$$\frac{P_2}{P_e} = \frac{2k}{k+1} M_e^2 \sin^2\beta - \frac{k-1}{k+1}, \tag{5}$$

$$\frac{\rho_e}{\rho_2} = \frac{2}{k-1}\left(\frac{1}{M_e^2 \sin^2\beta} + \frac{k-1}{2}\right), \tag{6}$$

$$M_2^2 \sin^2(\beta - \theta_e - \varphi) = \left(1 + \frac{k-1}{2} M_e^2 \sin^2\beta\right) \Big/ \left(k M_e^2 \sin^2\beta - \frac{k-1}{2}\right). \tag{7}$$

Taking the parameters behind the oblique shock as the parameters of the undisturbed stream in Prandtl-Meyer flow, we determine the angle of spreading of the jet over the obstacle surface [2]:

$$\theta_s = \omega(M_{e.s}) - \omega(M_2), \tag{8}$$

where the Mach number at the jet boundary in the plane of the obstacle is determined from the relation

$$\frac{\pi(M_2)}{\pi(M_{e.s})} = \frac{P_{02} P_2}{P_e P_{01}} = \frac{P_2}{P_e}. \tag{9}$$

Figure 1 shows the results for spreading angle as calculated by the above method, compared with the experimental data.

To calculate the static pressure distribution over the obstacle in the interaction region, we assume that, in the plane of the obstacle, discharge of the supersonic jet occurs from some fictitious nozzle (of diameter d_f) with a uniform distribution of parameters at its exit section. The parameters at the nozzle exit are assumed to be those behind the oblique shock. The diameter of the fictitious nozzle, d_f, is found from the following reasoning. The lines of interaction of the jet with the obstacle in the vicinity of the initial interaction point may be determined as lines of intersection of the surface of the jet with the plane of the obstacle. Then, starting from certain points on the interaction lines, spreading out of the jet takes place in the plane of the obstacle inside an angle of $2\theta_s$ (Fig. 2).

Therefore, at these points an interaction line is tangent to the sides of angle $2\theta_s$. The distance between the points of tangency should be assumed to be the diameter of the fictitious nozzle.

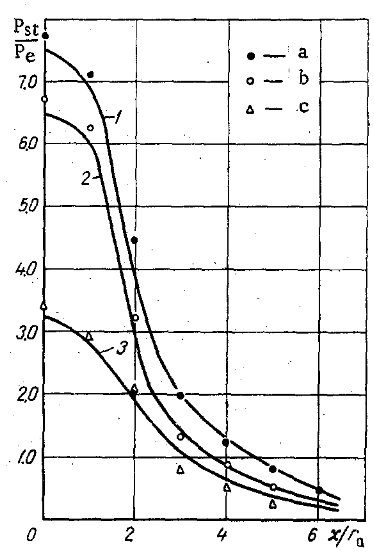

Fig. 4. Distribution of static pressure over the obstacle: 1 and a—theory and experiment, respectively, for a jet with initial parameters $M_a = 3.5$, n = 1.0 and $\varphi = 30°$; 2 and b—the same, for $M_a = 3.0$, n = = 1.0 and $\varphi = 30°$; 3 and c—$M_a = 3.0$, n = 1.0 and $\varphi = 20°$.

We shall examine discharge of gas from a nozzle, when the off-design factor is n = 1. The simplicity of this case is firstly, that the boundary of the jet is given by the equation of a cylinder (whence it is clear that the line of interaction of the jet with the obstacle in a certain neighborhood of the point O will be an ellipse), and secondly, that the gas parameters are uniform at the nozzle exit.

We shall write down the equation for the free surface of the jet in a coordinate system associated with the jet axis (Fig. 2):

$$(y')^2 + (z')^2 = R^2. \tag{10}$$

The equation of the obstacle plane, which is at a distance s_0 from the nozzle exit along the axis O'x', has the form, in this coordinate system,

$$(x' - s_0)\,\mathrm{tg}\,\varphi = y'. \tag{11}$$

Solving Eqs. (10) and (11) simultaneously, we find the equation of an ellipse:

$$(z')^2 + (x' - s_0)\,\mathrm{tg}^2\,\varphi = R^2. \tag{12}$$

We shall go over to a coordinate system associated with the obstacle. The conversion formulas, as may be seen from Fig. 2, have the form

$$x' = \left(x - \frac{R}{\sin\varphi}\right)\cos\varphi + s_0,$$
$$y' = y\cos\varphi - R,$$
$$z' = z. \tag{13}$$

Simultaneous solution of (10) and (11) in the new coordinate system gives

$$z^2 + \left(x - \frac{R}{\sin\varphi}\right)^2 \sin^2\varphi = R^2. \tag{14}$$

Going over to the dimensionless quantities $\bar{z} = z/R$ and $\bar{x} = x/R$, we obtain

$$\bar{z}^2 + \left(\bar{x} - \frac{1}{\sin\varphi}\right)^2 \sin^2\varphi = 1. \tag{15}$$

Since spreading out of the stream takes place within an angle $2\theta_s$, the equation of the tangent to the interaction line (an ellipse), making this angle, has the form

$$\frac{\bar{z}}{\sin\varphi\sqrt{1-\bar{z}^2}} = \mathrm{ctg}\,\theta_s. \tag{16}$$

The last equation also determines the diameter of the fictitious nozzle ($\bar{d}_f = 2\bar{z}$):

$$\bar{d}_f = \frac{2\sin\varphi}{\sqrt{\mathrm{tg}^2\,\theta_s + \sin^2\varphi}}. \tag{17}$$

Figure 3 shows a graph of the function (17) for a certain range of variation of the parameters φ and θ_s.

We shall calculate the parameters of the gas spreading out over the obstacle. Knowing the diameter of the fictitious nozzle, and also the initial parameters of the discharge at its exit section (the parameters behind the oblique shock, found from formulas (4)–(7)), we determine the pressure and the Mach number at the obstacle, following the usual method of calculation for free jets [2].

Expansion of the flow takes place in a pencil of expansion waves. The Mach angles of the boundary waves are determined by the formula

$$\alpha = \arcsin\frac{1}{M}, \tag{18}$$

where $M = M_f$ is taken for the initial wave, and $M = M_e$ for the final wave.

Later on, in calculating the gas parameters downstream in the plane of the obstacle, we shall divide the pencil of expansion waves into a finite number of waves of equal intensity, i.e., if the stream is turned by an angle θ_s in going through the whole pencil of waves, then in going through an individual wave, the stream is turned through an angle θ_s/n. We can obtain a relationship between the angle through which the flow is turned by an expansion wave and its Mach number, if we integrate the equation of the characteristics, written in the hodograph plane, lying on the obstacle:

$$d\theta \pm \frac{\mathrm{ctg}\,\alpha}{v}\,dv = 0. \tag{19}$$

To construct an expansion wave with number $i = 1, 2, \ldots, n$, we must plot the mean angle $\bar{\alpha}$, found from the Mach numbers before and after the wave,

$$\bar{\alpha} = \arcsin\frac{2}{M_i + M_{i+1}} \tag{20}$$

from the mean flow direction

$$\bar{\theta} = \frac{\theta_i + \theta_{i+1}}{2}. \tag{21}$$

The static pressures on the expansion wave characteristics are determined from the formula

$$\frac{P_i}{P_{i-1}} = \left[\left(1 + \frac{k-1}{2}M_{i-1}^2\right) \times \right.$$
$$\left. \times \left(1 + \frac{k-1}{2}M_i^2\right)\right]^{\frac{k}{k-1}}. \tag{22}$$

Figure 4 shows a graph comparing experimental values with the pressure distribution along the axis of symmetry of the gas flow over the obstacle (Ox), obtained by calculation as described above. For each case the error does not exceed 5%.

NOTATION

M is the Mach number; P is the static pressure; ρ is the density; v is the flow velocity; n is the off-design factor for the jet; k is the adiabatic exponent; φ is the angle of inclination of the obstacle to the jet axis; ϑ_a is the half-angle of the nozzle; θ is the angle of inclination of the jet boundary; α is the Mach angle; β is the angle of inclination of the shock; s is the distance from the nozzle exit to the obstacle along the jet axis; d is the diameter of the exit section of the nozzle; $\bar{d}$ is the dimensionless diameter of the nozzle exit section, referred to the exit radius of the actual nozzle; $\omega(M)$ is the Prandtl-Meyer function. All dimensionless linear quantities are referred to the radius of the exit section of the actual nozzle. Subscripts: a) parameters at the exit of the actual nozzle; e) parameters at the flow boundary; f) parameters at the exit of the fictitious nozzle; s) parameters relating to spreading of the flow over the obstacle; 2) parameters of the gas behind the inclined shock.

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